Multicell Coordinated Scheduling with Multiuser ZF Beamforming: Policies and Performance Bounds

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Abstract—We consider a coordinated multiuser scheduling problem for a multicell mutually interfering broadcast network. In particular, we focus on a two-cell cluster, where both base stations have only local data and local channel state information, but each has sufficient number of antennas to serve multiple homogeneous users under a full zero-forcing beamforming transmission. The scheduling problem is formulated as finding proper scheduled users and hence beamformers across the cells such that the sum rate is maximized. We uncover the structure for a good scheduling decision, which in turn motivates three distributed coordinated scheduling policies of different levels of complexity. For the simplest policy, we derive a lower bound on the expected achievable sum rate. It is shown in the large user population limit, the simplest policy suffices to preserve the best possible multiplexing gain and multiuser diversity gain for the model studied, but it does induce a pairing loss on the sum rate due to the limited coordination between cells.

I. INTRODUCTION

Finding the optimal user scheduling decision in a cellular network is normally known as a hard problem because of its inherent combinatorial property. Previously, the scheduling problem has been addressed for single-cell broadcast downlink with transmit multiuser beamforming, such as zero-forcing beamforming (ZF-BF). In particular, it was shown in [1] that the ZF-BF, in conjunction with a low-complexity Semiorthogonal User Selection (SUS) algorithm, achieves the same asymptotic sum rate as dirty-paper coding. Similar scheduling problems have also been studied in the literature accounting for user heterogeneity or imperfect channel state information (CSI), see, e.g., [2] and [3].

In this work, we consider a multicell network with the full frequency reuse desired for current system designs [4], [5]. As in Fig. 1, we consider a two-cell setup: each cell consists of one 2\textsuperscript{M}-antenna base station (BS) and K single-antenna homogeneous user equipments (UEs), where \( K \geq M \). Each UE is served by the BS in its anchor cell, and each BS only has data intended for its serving UEs. With regards to CSI, each BS only knows the local CSI originating from itself to all UEs in the network. A finite-capacity backhaul connects the BSs to allow certain information exchange for coordination, but not to be extended of data sharing.

This setup is referred to as a mutually interfering broadcast network in [6]. According to [6, Theorem 2], the maximum achievable degrees of freedom (DoFs) are 2\textsuperscript{M} in total for the network studied here, if the transmission is restricted to some practical single-shot beamforming scheme such as ZF-BF and no symbol extension is allowed. Consider a symmetric system, in which an even allocation of M DoFs to each cell is made, with M out of K UEs per cell co-scheduled for transmission in a time slot. Under this design choice, the BSs are able to zero-force both intra- and inter-cell interference even though each is provided only local data and local CSIs. We term this scheme as the full ZF-BF transmission scheme. It is clear that the exact ZF beamformer generation at one cell is coupled with the scheduling decision in the other cell.

Our results are as follows. First, we formulate the coordinated scheduling problem with an objective of maximizing the network sum rate. We uncover the structure for a good scheduling decision and develop three distributed scheduling policies, each of which involves single-cell SUS and requires different levels of message passing. These policies are of lower computational complexity and consume less communication overhead than a global optimization scheduling via exhaustive search. Second, a theoretical lower bound on the expected sum rate is derived for the simplest policy. An important observation is that in the limit of large K, the simplest policy suffices to preserve the best possible multiplexing gain and multiuser diversity gain, but with a pairing loss on the expected sum-rate due to limited coordination between cells.

Notation: Boldface uppercase and lowercase letters are used to denote matrices and vectors, respectively. \( \mathcal{CN}(0, \sigma^2) \) denotes a zero-mean complex Gaussian distribution with variance \( \sigma^2 \). \( (.)^\dagger \), \( E(.) \) and \( tr(.) \) denote the conjugate transpose, expectation and trace operation, respectively. \( [a]_\ast \) stands for the \( L_2 \) norm of vector \( a \) and \( I_M \) is an \( M \times M \) identity matrix.

II. MODEL NOTATION AND PROBLEM FORMULATION

A. System Model Notation

In the system, denote the BS in cell \( i \) as BS \( i \), the \( k \)th UE in cell \( i \) as UE\textsuperscript{(i)}\textsubscript{k} and the set of scheduled UE indices in cell \( i \) as \( A^{(i)} \), thus \( A^{(i)} \subset \{1, \ldots, K\} \) with \( |A^{(i)}| = M \), \( i = 1, 2 \).

Let \( u_k^{(i)} \sim \mathcal{CN}(0, 1) \) be the data symbol intended for UE\textsuperscript{(i)}\textsubscript{k}, and let unit-norm vector \( w_k^{(i)} \in \mathbb{C}^{2M\times1} \) be a linear ZF beamformer associated with symbol \( u_k^{(i)} \). Then the transmitted signal vector at BS \( i \) is given by

\[ s^{(i)} = \sum_{k \in A^{(i)}} w_k^{(i)} \sqrt{P_k^{(i)}} u_k^{(i)}, \]  

(1)

where \( P_k^{(i)} \) is the transmission power allocated for UE\textsuperscript{(i)}\textsubscript{k}, subject to a per-BS power constraint as \( tr[E[s^{(i)}s^{(i)\dagger}]]=... \)
\[ \sum_{k \in A_i} P^{(i)}_{k} \leq P. \] The received signal at UE_{k}^{(i)} is given by
\[ y_{k}^{(i)} = h_{k}^{(i)} s_{k}^{(i)} + \sqrt{\alpha_{k}} f_{k}^{(i)} s_{k}^{(i)} + z_{k}^{(i)}, \]
where notation \( i \) denotes the complement of \( i \) with respect to the set \{1, 2\}; \( h_{k}^{(i)} \in \mathbb{C}^{1 \times 2M} \) stands for the intra-cell small-scale fading channel between BS \( i \) and UE_{k}^{(i)}, with i.i.d. entries \( \sim \mathcal{CN}(0,1) \); \( f_{k}^{(i)} \) \( \in \mathbb{C}^{1 \times 2M} \) stands for the inter-cell small-scale fading channel between BS \( i \) and UE_{k}^{(i)}, also with i.i.d. entries \( \sim \mathcal{CN}(0,1) \); \( z_{k}^{(i)} \sim \mathcal{CN}(0, N_0) \) is the random noise at UE_{k}^{(i)}; and \( \alpha \) represents the inter-cell interference strength.

Under the full ZF-BF transmission scheme, beamformers for each cell are selected such that they satisfy conditions:
\[ \begin{align*}
    h_{j}^{(i)} w_{k}^{(i)} &= 0, \quad \forall j \in A(i) \setminus k, \quad (3) \\
    f_{l}^{(i)} w_{k}^{(i)} &= 0, \quad \forall l \in A(i). \quad (4)
\end{align*} \]
Let \( H^{(i)} \) be a channel matrix whose rows correspond to a collection of intra-cell channels from BS \( i \) to the active UEs in its cell \( A(i) \) and inter-cell small-fading channels from BS \( i \) to the active UEs in its neighboring cell \( A(i) \):
\[ H^{(i)} = \begin{bmatrix}
    h_{1}^{(i)(1)} & \cdots & h_{M}^{(i)(1)} \\
    h_{1}^{(i)(2)} & \cdots & h_{M}^{(i)(2)} \\
\end{bmatrix}, \]
and let \( W^{(i)} \) denote the beamforming matrix whose columns correspond to a set of unit-norm beamformers for cell \( i \):
\[ W^{(i)} = \begin{bmatrix}
    w_{1}^{(i)(1)} & \cdots & w_{M}^{(i)(1)} \\
    w_{1}^{(i)(2)} & \cdots & w_{M}^{(i)(2)} \\
\end{bmatrix}. \quad (5) \]
A standard choice of \( W^{(i)} \) that satisfies (3)-(4) is given by
\[ W^{(i)} = \hat{W}^{(i)} \quad (6) \]
where \( \hat{W}^{(i)} \) is a \( 2M \times 2M \) square matrix formed via
\[ \hat{W}^{(i)} = H^{(i)} (H^{(i)} H^{(i)})^{-1} \text{diag} \left( \sqrt{\gamma_{1}^{(i)}}, \ldots, \sqrt{\gamma_{2M}^{(i)}} \right), \]
with parameter \( \gamma_{l}^{(i)} \) defined as
\[ \gamma_{l}^{(i)} = \frac{1}{\| H^{(i)} H^{(i)} \|_{l,l}} - 1, \quad l = 1, \ldots, 2M. \quad (7) \]
Here, \( \gamma_{l}^{(i)} \) can be viewed as the effective beamforming gain associated with the \( l \)th beamformer for UE_{k}^{(i)} \( \in A(i) \), \( i = 1, \ldots, M \).

B. Scheduling Problem Formulation
We formulate a coordinated scheduling problem of maximizing the sum rate for the setup studied as follows:
\[ \mathcal{P} : \max_{A(i) \subseteq \{1, 2\}, R \geq \sum_{i=1}^{2} R^{(i)}, (8) \]
where the individual cell-rate \( R^{(i)} \) is quantified by
\[ R^{(i)} = \sum_{k=1}^{M} \log_2 \left( 1 + \gamma_{k}^{(i)} P^{(i)}_{A(i)(k)} / N_0 \right). \quad (9) \]
In (9), \( \gamma_{k}^{(i)} \) is adopted from (7) and the optimal power allocations \( \{ P^{(i)}_{A(i)(k)} \} \) are found independently at BS \( i \) by the conventional water-filling strategy:
\[ P^{(i)}_{A(i)(k)} = \left( \lambda - N_0 / \sqrt{\gamma_{k}^{(i)}} \right)^+, \quad (10) \]
with \((x)^+\) denoting \( \max\{x, 0\}\), and the water level \( \lambda \) chosen to satisfy \( \sum_{k=1}^{M} P_{A(i)(k)} = P \), for \( i = 1, 2 \).

Fig. 1. A two-cell broadcast network with coordinated scheduling and ZF-BF.

It is straightforward to solve problem \( \mathcal{P} \) via exhaustive search, leading to a globally optimized scheduling (GOS). Despite its optimality, this approach has drawbacks. First, it relies on a central processor that is able to collect the network-wide information (e.g., CSIs) to perform the search. Second, it has a search space of size \( \left( \begin{array}{c} M \\ 2 \end{array} \right) \), which can be significantly large for even moderate population \( K \). We are thus motivated to develop some low-complexity scheduling policies that can be implemented in a distributed manner with limited message passing and with small performance loss.

III. SCHEDULING POLICIES
A. Structure for A Good Scheduling Decision
We first attempt to understand the structure for a potentially good scheduling decision. It is noted from (9) that the dominant quantities in the rate evaluation are the effective beamforming gains \( \gamma_{k}^{(i)} \). Therefore, it is of interest to understand how a scheduling decision influences the resulting gains.
To do so, let us consider a fixed admissible scheduling decision \( A = A(1) \times A(2) \). In each active set given, for ease of exposition, we order the UEs according to their original indices in an ascending manner and then re-label them with \( \{1, \ldots, M\} \); we also ignore all cell indices due to the cell symmetry under the homogeneous setup. Thus we have intra-cell channels \( \{h_{1}, \ldots, h_{M}\} \) and inter-cell channels \( \{f_{1}, \ldots, f_{M}\} \) in channel matrix for each cell. Furthermore, let \( \{g_{1}, \ldots, g_{M}\} \) be the set of orthogonal vectors that define the space spanned by the intra-cell channels with
\[ g_{1} = h_{1}; \quad g_{k} = h_{k} \left( I_{M} - \sum_{j=1}^{k-1} g_{j} g_{j}^{\dagger} / \| g_{j} \|^{2} \right), \quad k \in [2, M]. \quad (11) \]
In this way, the channel matrix \( H \) is decomposed as
\[ H = \begin{bmatrix}
    D_{1} & 0_{M} \\
    0_{M} & D_{2} \\
\end{bmatrix} \begin{bmatrix}
    R_{1} & 0_{M} \\
    0_{M} & R_{2} \\
\end{bmatrix} \begin{bmatrix}
    Q_{1} & \cdots & Q_{2} \\
\end{bmatrix}, \quad (12) \]
where bottom-right block matrix \( R_{2} = I_{M} \), matrices \( D_{2} \) and \( Q_{2} \) relate to inter-cell interference channels only with \( D_{2} = \text{diag}(\| f_{1} \|, \ldots, \| f_{M} \|) \) and
\[ Q_{2} = \begin{bmatrix}
    \tilde{f}_{1} \\
    \| \tilde{f}_{1} \| \\
    \vdots \\
    \| \tilde{f}_{M} \| \\
\end{bmatrix} \begin{bmatrix}
    \tilde{f}_{1} \\
    \| \tilde{f}_{M} \| \\
\end{bmatrix}, \quad (13) \]
while top-left block matrices $D_1$, $R_1$ and $Q_1$ are functions of intra-cell channels, whose definitions are given as follows:

$$D_1 = \text{diag}(\|g_1\|, \ldots, \|g_M\|),$$

$$R_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \epsilon_{2,1} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \epsilon_{M,1} & \cdots & \cdots & 0 \end{bmatrix},$$

and $Q_1 = \begin{bmatrix} \frac{g_1}{\|g_1\|} \\ \frac{g_2}{\|g_2\|} \\ \vdots \\ \frac{g_M}{\|g_M\|} \end{bmatrix}.$

Now, the effective beamforming gains are calculated as

$$\gamma_k = \frac{1}{\|x_k\|^2 / \|x_k\|^2 (1 + \|x_k\|^2 (BE^{-1}B^\dagger) x_k^\dagger / \|x_k\|^2)} = \frac{\|x_k\|^2 (1 + \|x_k\|^2 (BE^{-1}B^\dagger) x_k^\dagger / \|x_k\|^2)}{\|x_k\|^2} \frac{1}{\|x_k\|^2 / \|x_k\|^2},$$

where $\epsilon_{i,j} = \frac{h_{i,j}}{\|h_i\|^2 \|h_j\|^2}$.

Observe that each beamforming gain $\gamma_k$ consists of two components. In particular, the first component $\gamma_1$ depends on the intra-cell channels only, and thus it can solely be influenced by its own cell’s scheduling decision. Given this fact, we term this component as the Single-Cell-Scheduling Gain (SCSG) for each active UE. On the other hand, the second component $\frac{1}{F_k}$ is a function of both the intra- and inter-cell channels, and therefore it can be instead influenced by both cell’s scheduled UEs. Moreover, since $F_k \geq 1$, $\frac{1}{F_k}$ could not be larger than unity and thus introduces a penalty for the effective beamforming gain. For this reason, we term $F_k$ as the Multicell-Multiuser-Pairing Penalty (MMPP) for each active UE. There is clearly a tradeoff between the SCSG and the MMPP, and a good scheduling policy will judiciously select UEs taking into account this tradeoff.

### B. Low-Complexity Distributed Scheduling Policies

We are now ready to outline several low-complexity scheduling policies. Note that in prior work [7], we have proposed the first two policies for a heterogeneous network, but without the rigorous justification provided here.

1) **Policy 1 (Blind SUS):** In the first policy, each BS forms the scheduling user set $A^{(i)}$ independently based on the SUS algorithm [1] and then informs its partner of the active UE indices via the backhaul link. The ZF-BF as in (6) is then performed at each BS according to the scheduling decisions.

This policy mainly leverages the SUS algorithm to maintain large SCSGs for UEs scheduled in each cell. It possesses a search space of size at most $\sum_{j=0}^{M-1} (K - j)$ at each BS for the SUS, and involves only $M$ UE indices exchange between the BSs along each direction. The key disadvantage, however, is that it does not control the MMPP loss at all.

2) **Policy 2 (Follower-Leader):** In the second policy, the two BSs alternately behave as a leader and then a follower on a slot by slot basis and some additional information is exchanged. Specifically, in the initial phase, each BS independently forms a scheduled candidate list $C^{(i)}$ that consists of $M'$ UEs (with $M < M' \ll K$) based on the SUS. The follower, e.g., BS $i$, forwards its list $C^{(i)}$ to the leader, BS $j$, via the backhaul link. In the pairing phase, the leader BS finds the best scheduling combination $A^*$ among $A = \{A^{(1)} \times A^{(2)} : A^{(2)} \in C^{(1)}; A^{(2)} \subseteq C^{(2)}\}$ maximizing its own cell-rate. The decision $A^*$ is then informed to the follower. This policy allows the leader to balance its SCSG and MMPP.

3) **Policy 3 (Joint Selection):** The third policy further improves upon the second by introducing slightly more information exchange. Specifically, in the initial phase, each BS creates a list $C^{(i)}$ that consists of $M'$ UEs ($M < M' \ll K$) based on the SUS algorithm and it sends the list to its partner. Upon receiving the proposal, each BS evaluates its own cell-rate for all combinations in $A$. To jointly control both cell’s SCSG and MMPP and find the best scheduling combination, one BS, e.g., BS $i$, passes its own cell-rates for $A$ to its partner BS $j$. The latter is then able to evaluate the network sum-rate accordingly for each combination in $A$ and informs BS $i$ about the final scheduling decision with the largest sum-rate.

It is clear that an enhanced coordination that potentially boosts network throughput is performed in this policy. This policy has the same complexity of forming the candidate list for the initial phase and the same search space at the pairing phase as in the second. But it needs to compute candidate-rates at both cells and requires larger backhaul capacity to exchange the additional candidate list and candidate-rates necessary.

### IV. PERFORMANCE BOUNDS

In this section, we derive performance bounds on the scheduling policies. We are particularly interested in the metric expected sum-rate, since this metric naturally represents the long-term system throughput for the network studied.

We first propose an upper bound on the expected sum-rate achieved by the GOS. This bound is thus also an upper bound for all the distributed policies proposed.
Proposition 1: Let $P_{avg}^{GOS}$ denote the expected sum-rate achieved by the GOS. Then we have

$$ P_{avg}^{GOS} \leq 2M \cdot \log_2 \left( 1 + \frac{P}{MN_0} \left( \ln K + 2M \ln \ln K + O \left( \ln \ln \ln K \right) \right) \right). $$

Proof: The performance of our setup is dominated by an enhanced two-cell network where each cell is isolated and independently schedules $M$ UEs with orthogonal channel vectors all tied for the maximum norm $\|h_{1}(1)\|^2$. The result here then follows by deriving a performance bound on the enhanced system using extreme value theory [8].

It is immediate to see that, for the network studied, the best possible multiplexing gain is $2M$ and the multiuser diversity gain roughly scales as $\ln \ln K$.

Next, we focus on lower bounds on the performance of the distributed coordinated policies. In particular, we derive a lower bound on the expected sum-rate attained by the simplest policy, i.e., Blind SUS. This lower bound thus serves as a worst-performance prediction for the system. We first introduce a uniform bound on the expected MMPP for each UE scheduled under the Blind SUS.

Lemma 1: Let $U_B \in \mathbb{C}^{M \times M}$ denote an arbitrary unitary matrix. Then, matrix $U_B^\dagger (B^{-1}B^\dagger)U_B^\dagger$ has the same statistics as matrix $B^{-1}B^\dagger$ under the Blind SUS.

Proof: In the Blind SUS, neither cell has any control over the subset of UEs scheduled in the other cell. Therefore, under any scheduling realization, all inter-cell channels $F = \{f_1, \ldots, f_M\}$ are independent Rayleigh-distributed random vectors and thus matrix $Q_2$ (see (13)) can be viewed as a random matrix consisting of $M$ i.i.d. isotropic row vectors $\{\tilde{r}_1, \ldots, \tilde{r}_M\}$ on the $2M$-dimensional unit sphere. On the other hand, $Q_1$ consists of orthonormal basis vectors and is generated according to the intra-cell channels $H = \{h_1, \ldots, h_M\}$ for the active UEs selected by the SUS. It is clear that $Q_1$ and $Q_2$ are independent. Once $Q_1$ is formed, without loss of generality, one can always choose coordinates so that $Q_1 = \left[ I_M \mid \mathbf{0}_M \right]$. With the new coordinates, we have:

$$ B = Q_1Q_2^\dagger = \left[ I_M \mid \mathbf{0}_M \right] \left[ \begin{array}{c} Q_{21}^\dagger \\ Q_{22}^\dagger \end{array} \right] = Q_{21}^\dagger, \quad (23) $$

$$ E = C - B^\dagger B = Q_{22}Q_{22}^\dagger, \quad (24) $$

where we have represented $Q_2$ as $Q_2 = \left[ Q_{21} \mid Q_{22} \right]$. Let $U_B \in \mathbb{C}^{M \times M}$ be an arbitrary unitary matrix. Then we can construct a new unitary matrix $U$ in form of

$$ U = \left[ \begin{array}{cc} U_B & \mathbf{0}_M \\ \mathbf{0}_M & I_M \end{array} \right]. \quad (25) $$

Applying $U$ on $Q_2^\dagger$ yields:

$$ UQ_2^\dagger = \left[ \begin{array}{c} U_BQ_{21}^\dagger \\ U_BQ_{22}^\dagger \end{array} \right] = \left[ \begin{array}{c} U_BB \\ U_BQ_{22}^\dagger \end{array} \right]. \quad (26) $$

and the objective $B^{-1}B^\dagger$ becomes $(U_BB)B^{-1}(U_BB)^\dagger$. Since $U_BB$ has the same statistics as $B$ and $E = Q_{22}Q_{22}^\dagger$ is unchanged, we have therefore proved the lemma.

With this crucial lemma, we proceed to propose a uniform bound on the expected pairing penalty for each scheduled UE.

Proposition 2: Under the Blind SUS, the expected MMPP (in log-scale), i.e., $E[\log_2(B_k)]$, is upper bounded as

$$ E[\log_2(B_k)] \leq E_r \left[ \log_2 \left( 1 + \frac{T}{M} \right) \right] = F_{avg}, \quad (27) $$

where $T = tr(B^{-1}B^\dagger)$. Proof:

$$ E_{h,F}[\log_2(B_k)] = E_{h,F} \left[ \log_2 \left( 1 + \tilde{x}_k (B^{-1}B^\dagger) \tilde{x}_k^\dagger \right) \right], \quad (28) $$

$$ = E_{h,F,U_B} \left[ \log_2 \left( 1 + \tilde{x}_k (B^{-1}B^\dagger) U_B \tilde{x}_k^\dagger \right) \right], \quad (29) $$

$$ = E_{h,F,U_B} \left[ \log_2 \left( 1 + \tilde{x}_k (B^{-1}B^\dagger) U_B \tilde{x}_k^\dagger \right) \right] \leq \log_2 \left( 1 + \frac{M}{k} \sum_{k=1}^M |x_{k,1}|^2 \lambda_k \right) \quad (30) $$

$$ \leq E_{(\lambda_k)_{k=1}^M,U_B} \left[ \log_2 \left( 1 + \frac{1}{M} \sum_{k=1}^M \lambda_k \right) \right] \quad (31) $$

$$ = E_{T} \left[ \log_2 \left( 1 + \frac{T}{M} \right) \right] = F_{avg}, \quad (35) $$

where

- in (30), we introduce $U_B$ as a uniformly distributed random unitary matrix (denoted as $U_B \sim U(M)$ for short) and use the conclusion in Lemma 1;

- to obtain (31), first rewrite $U_B^\dagger (B^{-1}B^\dagger)U_B = (VU_B)^\dagger \text{diag}(\lambda_1, \ldots, \lambda_M) (VU_B)$ as a result of the singular value decomposition on Hermitian matrix $(B^{-1}B^\dagger)$, where $\{\lambda_k\}_{k=1}^M$ denote its eigenvalues arranged in a descending order; with $\{\lambda_k\}_{k=1}^M$ fixed, now observe that matrix $U_B \text{diag}(\lambda_1, \ldots, \lambda_M) U_B^\dagger$ has the same statistics as $(VU_B)^\dagger \text{diag}(\lambda_1, \ldots, \lambda_M) (VU_B)$ since $U_B \sim U(M)$;

- to obtain (32), note $\tilde{x}_k$ is a unit-norm row vector and without loss of optimality, we can assume $\tilde{x}_k = [1 \ 0 \ \cdots \ 0]$. This would not change the statistics since one could view $U_B$ as an arbitrary rotation applied to $\tilde{x}_k$ and the rotation is uniformly distributed;

- (33) follows from Jensen’s inequality;

- (34) holds due to $E[U_B | x_{k,1}] = \frac{1}{M} I_M \forall k$, under the assumption that $U_B \sim U(M)$;

- in (35), we define $T = \sum_{k=1}^M \lambda_k = tr(B^{-1}B^\dagger)$.

Note that finding the exact distribution of $T$ is very challenging in general. But one can numerically compute the quantity $F_{avg}$ by generating appropriate samples of variables involved in the characterization. As discussed in Lemma 1, without loss of optimality, one can always choose the coordinates so that $Q_1 = \left[ I_M \mid \mathbf{0}_M \right]$ and generate $Q_2$ as a collection
of $M$ i.i.d. isotropic row vectors on the $2M$-dimensional unit sphere. Therefore, this quantity is independent of the exact scheduled UEs and also independent of the system user population $K$ under the Blind SUS. We also remark that in the case of $M = 2$, we derive an alternative characterization for the expected MMPP upper bound, which appears to be relatively easier to compute. This result is stated as a corollary of Proposition 2 whose proof is omitted.

**Corollary 1:** In the case of $M = 2$, we further have

$$F_{\text{avg}} \leq E_{V_1, V_2, W} \left[ \log_2 (V_1 + V_2 - 2V_1V_2W) \right] + \frac{8}{3 \ln 2} - 1,$$

where independent variables $V_1, V_2 \sim \text{median}(U_1, U_2, U_3)$ with $U_k \sim \text{Unif}[0, 1]$, and $W \sim \text{Unif}[0, 1]$.

We now formalize a lower bound on the Blind SUS rate.

**Proposition 3:** Let $R_{\text{avg}}$ denote the expected sum-rate achieved by the Blind SUS. Then as $K \to \infty$, we have that

$$R_{\text{avg}}^{\text{Blind}} \geq 2M \left( \log_2 \left( \frac{P}{MN_0} \ln K - F_{\text{avg}} \right) + \gamma_0 \right).$$

**Proof:** We start with the definition of $R_{\text{avg}}^{\text{Blind}}$ as follows:

$$R_{\text{avg}}^{\text{Blind}} = \sum_{i=1}^{2} \sum_{k=1}^{M} \mathbb{E} \left[ \log_2 \left( 1 + \frac{P}{MN_0} \gamma_k \right) \right]$$

$$\geq 2 \sum_{k=1}^{M} \left( \mathbb{E} \left[ \log_2 \left( 1 + \frac{P}{MN_0} \gamma_k \right) \right] + \mathbb{E} \left[ \log_2 \left( \frac{1}{F_k} \right) \right] \right)^{+}$$

$$\geq 2M \left( \log_2 \left( \frac{P}{MN_0} \ln K - F_{\text{avg}} \right) + \gamma_0 \right)^{+},$$

where (38) follows from the cell symmetry and the fact that the power allocation via water-filling strategy performs no worse than the equal power allocation strategy, and (39) follows from Proposition 2 and the asymptotic performance results for single-cell scheduling via the SUS as derived in [1, Theorem 1 and bounds (46)-(53)] as $K \to \infty$.

This lower bound indicates that in the limit of large $K$, the simplest policy suffices to preserve the best possible multiplexing gain and multiuser diversity gain for the network considered, but it does induce a pairing loss on the expected sum-rate due to the limited coordination between.

**V. NUMERICAL RESULTS**

We define $\text{SNR} = 10 \log_{10} \left( \frac{P}{\text{N}_0} \right)$ as the reference SNR (in dB) and vary SNR between $-5$ dB and $30$ dB for plotting. For each SNR, we calculates the expected (average) sum rate over 1000 channel realizations (transmission slots), under which small-fading channels are independently generated for UEs in both cells. Assume the total number of UEs within each cell is $K = 20$, and the number of antennas $2M = 4$ for each BS so that $M = 2$ UEs are co-scheduled for each cell.

Fig. 2 plots the expected sum rate as a function of SNR under the distributed scheduling policies and the GOS. In addition, upper bound of Proposition 1 and lower bound of (36) are also included. It is clear that the bounds we derived are valid with the expected rate attained by any considered policy sandwiched in between them. As expected, the Blind SUS performs the worst because of its lowest complexity. However, it is able to preserve roughly $80\%$ of the throughput attained by the GOS (e.g., at $\text{SNR} = 20$ dB), even though it only possesses $79\%$. With a little more complexity increase, the Follower-Leader policy improves upon the first one, while the Joint Selection offers the best performance that is close to the GOS (with $M' = 4$ in this example).

**VI. CONCLUDING REMARKS**

We have proposed three low-complexity coordinated scheduling policies for the two-cell setup considered. For the simplest policy, we have derived a lower bound on the expected achievable sum rate. Obtaining these results serves as an intermediate step towards a full understanding of the scheduling problem posed here. More could be explored for future work including assessing the asymptotic reduction of pairing loss by user selection and extension of this framework to a more general network, such as a multicell linear network.

**REFERENCES**


